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WORLDWIDE CORRELATIONS FOR SUBAERIAL AQUEOUS FLOWS WITH EXPONENTIAL LONGITUDINAL PROFILES

P. H. MORRIS* AND D. J. WILLIAMS
Department of Civil Engineering, The University of Queensland, Brisbane, QLD 4072, Australia

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ABSTRACT

It is shown that very strong, worldwide correlations exist between the bed concavity coefficients of a wide range of subaerial aqueous flows with exponential longitudinal profiles and both the corresponding stream segment lengths and exponential bed particle size diminution coefficients. The former correlation is complementary to an existing similar correlation for the exponential size diminution coefficients, while the latter is consistent with earlier theoretical correlations based on very limited data.

The data supporting the correlations extend over virtually the whole range of stream lengths, solids concentrations, and bed sediment particle sizes found on Earth. This universality strongly suggests that there are underlying mechanisms common to all kinds of mobile bed subaerial aqueous flows. However, the scatter of the data for the correlations is significant and is mostly attributable to variations in hydraulic conditions and sediment properties rather than measurement errors. Some of the conditions and properties have been identified, but others remain obscure. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: exponential profiles; hydraulic sorting; mobile bed flow; stream profiles

INTRODUCTION

It is well known that the exponential equation originally suggested by Schoklitsch (1937) for the longitudinal profiles of the middle Rhine, Maas, Enns and Mur Rivers applies to the profiles of many other natural streams (Shulits, 1941; Simons, 1977) and alluvial fans (Krumbein, 1937). It is also applicable to numerous laboratory flume data (Morris and Williams, 1996) and fine and coarse mine waste deltas (Williams and Morris, 1989; Morris and Williams, 1997a).

The exponential profile equation has also often been related to the well known exponential diminution in bed particle size in streams (Sternberg, 1875; Shulits, 1936; Schoklitsch, 1937). The latter phenomenon is known to apply to an even wider range of mobile bed subaerial aqueous flows (Morris and Williams, 1999) than the exponential profile equation.

Recently, it has been shown that a very strong worldwide correlation exists between exponential bed particle size diminution coefficients for mobile bed subaerial aqueous flows of all kinds and the lengths of the corresponding stream segments (Morris and Williams, 1999). Together, this correlation and the exponential profile and particle size diminution relationships suggest that a complementary worldwide correlation may exist for the bed concavity coefficients of such flows. In this paper, it is shown that worldwide correlations exist between these bed concavity coefficients and both the corresponding stream segment lengths and bed particle size diminution coefficients.

* Correspondence to: Dr P. H. Morris, Department of Civil Engineering, University of Queensland, Brisbane, Queensland 4072, Australia

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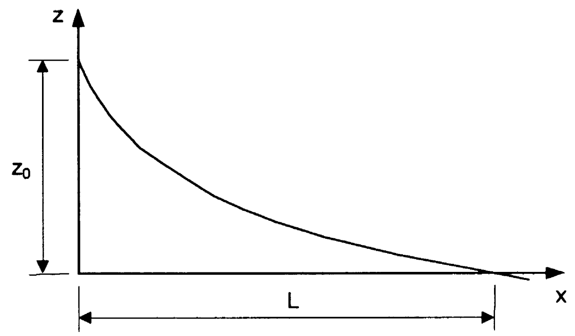

Figure 1. A diagrammatic representation of a stream segment with an exponential profile

STREAM PROFILE CORRELATIONS

Longitudinal stream profile equations

The change with increasing distance downstream of the slope of the bed (or the water surface) of many mobile bed subaerial aqueous flows is described by (Schoklitsch, 1937; Shulits, 1941):

$$S_x = S_0 \exp\left(-\varepsilon x\right) \tag{1}$$

where x is the longitudinal coordinate relative to an arbitrary datum, ε is a profile concavity coefficient characteristic of the given stream, and S_x and S_0 are the slopes of the bed at $x = x$ and $x = 0$, respectively.

Inverting Equation 1 gives, at $x = L$:

$$\varepsilon = \ln\left(\frac{S_0}{S_L}\right) L^{-1} \tag{2}$$

where L is the length of the stream or stream segment. The corresponding equations for the exponential bed particle size diminution coefficient, α, are given in Morris and Williams (1999).

For application to stream profile data expressed in terms of altitude rather than slope, Equation 1 can be converted to (Morris, 1993):

$$z_x = z_0 \frac{\exp(-\varepsilon x) - \exp(-\varepsilon L)}{1 - \exp(-\varepsilon L)} \tag{3}$$

where z_x is the height above the bed at $x = L$ (Figure 1).

Equations 1 to 3 apply to streams or stream segments flowing through their own sediments that have reached equilibrium under steady or quasi-steady hydraulic conditions (Morris and Williams, 1997b). Either hydraulic sorting or abrasion of bed sediment particles may be dominant; hence both slow aggradation and total sediment transport are permissible. The time scales for equilibrium may range from about 10^{-5} years for laboratory flume tests to 10^{6} years (Pizzuto, 1992) or more for total transport in natural streams. Flows may vary slowly or rapidly in time and space. However, in rapidly varying flows, the solids concentration by weight, C, and the flow Froude number are subject to constraints that ensure that the fluid and sediment flows are essentially mathematically independent (de Vries, 1973; Ribberink and van der Sande, 1984; Morris and Williams, 1997b,d). Also, stream segments undergoing significant subsidence, with significant lateral inflows of water or sediments, or with controls such as rock bars or dams, are excluded (Sinha and Parker, 1996; Pizzuto, 1992; Rice and Church, 1996; Morris and Williams, 1997b).

Worldwide longitudinal profile data

Data conforming to Equations 1 to 3 were obtained for 51 natural streams or stream segments, 25 alluvial fans and micro-fans, 30 mine tailings deltas, eight coal mine waste co-disposal deltas, and 20 laboratory flume tests (Table I). Data were accepted or rejected and L was determined using the same criteria as Morris and Williams (1999) for exponential sorting data. In most cases, the equations were fitted to the data using least-squares methods. The characteristic bed particle sizes, D, listed in Table I are median or, where indicated, maximum sizes. The level of significance, p, was known or was evaluated for 128 of the total of 137 data points.

The natural streams represented in the data set include rivers, creeks, arroyos, sandurs and glacial outwashes in North and South America, Japan, Australia, Great Britain and Europe. The alluvial fan data are from the United States, and the mine tailings data from South Africa, Canada, the United States and Australia.

Comparable worldwide bed particle diminution data are described in Morris and Williams (1999). The two data sets include both ε and α values for 39 natural streams or stream segments, 19 alluvial fans, three tailings deltas, eight co-disposal deltas, and one laboratory flume test. The p value is known for both ε and α for 64 of the 70 data points, but is unquantified for both ε and α for four of the points.

Worldwide correlation of ε *with L*

Figure 2a, a plot of ε versus L for the worldwide data, shows that they conform to:

$$\varepsilon = BL^{-1} \tag{4}$$

where B is a constant. The associated linear correlation coefficient, R, and p value are $-0{\cdot}980$ and less than 0·04 per cent, respectively, indicating a very strong correlation that extends over L values ranging from 1·1 m to 1770 km (Table I).

The upper and lower bounds of the data points are both well defined (Figure 2a). Expect for the three outlying points with L of about 10^{-3} km, the lower bound corresponds to the lower limit of the data points with fully quantified p values (Figure 2b). The isolated fourth outlier corresponds to the Arrow River Canyon alluvial fan (Table I).

The three neighbouring outliers were obtained from laboratory flume tests of very fine coal and gold mine tailings (Morris and Williams, 1996, 1997d, and unpublished data) that were duplicates or near-duplicates of tests which plot comfortably within the bounds. Large-scale plotting showed that there were very shallow convexities, about 1 mm or less in height, in the middle of each of the corresponding profiles that were probably attributable to small fluctuations in the discharge or C during the tests. The presence of these convexities in low concavity (less than 1 mm in some cases) bed profiles formed in very fine sediments deposited over short distances probably accounts for the abnormally low ε values obtained from these tests.

The width of the data band defined by the bounds is about 1·37 orders of magnitude, and, for ε and L expressed in m^{-1} and km, respectively, the geometric mid-range value of B is $1{\cdot}11 \times 10^{-3}$. If the four outliers are discounted, the values of R and p for the data set become $-0{\cdot}986$ and less than 0·04 per cent, respectively.

Given that no discordant data were arbitrarily discarded in the analysis, it is remarkable that the data set as a whole, which comprises a wide range of mobile bed subaerial aqueous flows with exponential profiles (Figure 2a), conforms to Equation 2, which applies to individual streams or stream segments. The corresponding L,α correlation obtained by Williams and Morris (1999) is directly comparable.

Homogeneity of the L,ε *data*

The distributions within the L,ε data band of the p for the individual points, of sand and gravel bed streams, and of normal (downstream fining) and reverse (downstream coarsening) sorting are shown in Figure 2b,c, and d, respectively. Here, as in Morris and Williams (1999), sand and gravel size bed particles are respectively defined as those smaller and larger than 2 mm in size. For the present data set, they comprise sand and silt down to 0·015 mm in size, and gravel, cobbles and boulders up to 0·63 m and 3·9 m in size for

Table I. Profile and bed particle size data for natural stream segments, alluvial fans, mine tailings and co-disposal deltas, and laboratory flumes

	Length (km)	D (mm)	S_0	ε (m^{-1})	p (%)	References
Natural streams						
Mississippi River 1932	1770·0	0·12–0·72	–	$1{\cdot}01 \times 10^{-6}$	–	Rafay in Simons (1977)
Mississippi River 1901	1158·0	sand	$1{\cdot}06 \times 10^{-4}$	$1{\cdot}04 \times 10^{-6}$	<0·04	Shulits (1941)
Ohio River	977·5	–	$1{\cdot}62 \times 10^{-4}$	$1{\cdot}29 \times 10^{-6}$	<0·04	Shulits (1941)
Peace River	469·9	15–50	$4{\cdot}59 \times 10^{-4}$	$1{\cdot}25 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
Peace River	410·1	0·14–0·35	$2{\cdot}05 \times 10^{-4}$	$3{\cdot}78 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
North Saskatchewan River	393·0	7–28	$5{\cdot}20 \times 10^{-4}$	$1{\cdot}42 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
Oldman River	282·4	16–39	$1{\cdot}42 \times 10^{-3}$	$3{\cdot}01 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
South Saskatchewan River	269·1	gravel	$5{\cdot}81 \times 10^{-4}$	$2{\cdot}49 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
Red Deer River	266·2	16–112	$2{\cdot}07 \times 10^{-3}$	$4{\cdot}81 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
Middle Rhine*	260·9	42–162	$1{\cdot}45 \times 10^{-3}$	$7{\cdot}71 \times 10^{-6}$	<0·04	Sternberg (1875); Woodford (1951)
North Saskatchewan River	258·6	21–130	$2{\cdot}43 \times 10^{-3}$	$2{\cdot}18 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
Rio Grande	241·4	0·14–0·50	–	$5{\cdot}72 \times 10^{-6}$	–	Rafay in Simons (1977)
Dunajec River*	192·0	45–555	$3{\cdot}92 \times 10^{-3}$	$1{\cdot}01 \times 10^{-5}$	<0·04	Unrug (1957)
Chapare River	176·2	gravel	$7{\cdot}35 \times 10^{-2}$	$2{\cdot}11 \times 10^{-5}$	<0·04	Bowman in Barrell (1925)
Rubicon River*	78·7	457–3290	$2{\cdot}31 \times 10^{-2}$	$2{\cdot}96 \times 10^{-5}$	<0·04	Scott and Gravlee (1968)
Kinu River	53·6	20–73	$6{\cdot}35 \times 10^{-3}$	$2{\cdot}55 \times 10^{-5}$	<0·04	Yatsu (1955)
Rapid Creek	48·0	5·1–24	$6{\cdot}80 \times 10^{-3}$	$2{\cdot}67 \times 10^{-5}$	<0·04	Plumley (1948)
Kinu River	46·0	0·41–0·86	$7{\cdot}74 \times 10^{-4}$	$2{\cdot}30 \times 10^{-5}$	2·5	Yatsu (1955)
Standing Stone Creek	43·5	12–117	$2{\cdot}04 \times 10^{-2}$	$8{\cdot}25 \times 10^{-5}$	<0·04	Brush (1961)
Nagara River	40·0	0·75–1·1	$1{\cdot}95 \times 10^{-4}$	$3{\cdot}19 \times 10^{-5}$	<0·04	Yatsu (1955)
Yahagi River	35·0	1·0–2·0	$1{\cdot}22 \times 10^{-2}$	$2{\cdot}55 \times 10^{-5}$	<0·04	Yatsu (1955)
Bear Butte Creek	33·8	3·7–52	$1{\cdot}42 \times 10^{-2}$	$5{\cdot}53 \times 10^{-5}$	<0·04	Plumley (1948)
Honey Creek	28·2	31–91	$2{\cdot}44 \times 10^{-2}$	$7{\cdot}81 \times 10^{-5}$	0·3	Brush (1961)
Battle Creek	28·0	5·3–33	$5{\cdot}76 \times 10^{-3}$	$2{\cdot}60 \times 10^{-5}$	3·1	Plumley (1948)
Slims River	26·0	–	$4{\cdot}76 \times 10^{-3}$	$1{\cdot}4 \times 10^{-4}$	–	Church (1972)
Knik River*	25·7	41–327	$2{\cdot}33 \times 10^{-3}$	$4{\cdot}97 \times 10^{-5}$	–	Bradley *et al.* (1972)
Abe River	23·0	17–83	$1{\cdot}12 \times 10^{-2}$	$4{\cdot}40 \times 10^{-5}$	<0·04	Yatsu (1955)
Watarase River	21·3	32–75	$1{\cdot}01 \times 10^{-2}$	$1{\cdot}06 \times 10^{-4}$	<0·04	Yatsu (1955)
Shaver Creek	20·6	23–141	$3{\cdot}18 \times 10^{-2}$	$1{\cdot}83 \times 10^{-4}$	<0·04	Brush (1961)
Sho River	20·0	29–48	$1{\cdot}11 \times 10^{-2}$	$6{\cdot}91 \times 10^{-5}$	<0·04	Yatsu (1955)
Gillis Falls	17·7	7·1–80	$1{\cdot}37 \times 10^{-2}$	$1{\cdot}10 \times 10^{-4}$	<0·04	Hack (1957)
Watarase River	15·8	0·37–0·84	$6{\cdot}87 \times 10^{-4}$	$3{\cdot}55 \times 10^{-5}$	0·6	Yatsu (1955)
King River	15·5	sand	$1{\cdot}41 \times 10^{-3}$	$1{\cdot}47 \times 10^{-4}$	<0·04	Locher (1997)
Hoffells Sandur	15·0	–	$2{\cdot}04 \times 10^{-2}$	$3{\cdot}4 \times 10^{-4}$	–	Church (1972)
North River	14·2	10–55	$9{\cdot}75 \times 10^{-3}$	$8{\cdot}59 \times 10^{-5}$	<0·04	Hack (1957)
Weiker Run	13·4	95–142	$3{\cdot}60 \times 10^{-2}$	$5{\cdot}01 \times 10^{-5}$	0·1	Brush (1961)
Nagara River	13·0	26–41	$1{\cdot}41 \times 10^{-3}$	$9{\cdot}99 \times 10^{-5}$	9·7	Yatsu (1955)
Upper Towey River	12·9	–	$2{\cdot}81 \times 10^{-2}$	$1{\cdot}25 \times 10^{-4}$	<0·04	Jones (1924)
Copper River Delta	12·0	30–300	$1{\cdot}55 \times 10^{-2}$	$1{\cdot}17 \times 10^{-4}$	–	Boothroyd (1970)
Joganji River	10·8	–	$1{\cdot}92 \times 10^{-2}$	$7{\cdot}89 \times 10^{-5}$	<0·04	Yatsu (1955)
Tye River	10·6	230–630	$8{\cdot}04 \times 10^{-2}$	$6{\cdot}48 \times 10^{-5}$	6·7	Hack (1957)
Sunwapta River	8·3	6·1–82	$1{\cdot}53 \times 10^{-2}$	$2{\cdot}75 \times 10^{-4}$	<0·04	Dawson (1988)
Joganji River	6·2	–	$4{\cdot}94 \times 10^{-2}$	$2{\cdot}72 \times 10^{-4}$	0·2	Yatsu (1955)
Reeds Run	5·2	30–86	$4{\cdot}16 \times 10^{-2}$	$2{\cdot}64 \times 10^{-4}$	<0·04	Brush (1961)
Pysgotwr River	3·9	–	$3{\cdot}29 \times 10^{-2}$	$3{\cdot}40 \times 10^{-4}$	<0·04	Jones (1924)
Arroyo Larguito	2·6	0·20–0·52	$7{\cdot}28 \times 10^{-2}$	$7{\cdot}83 \times 10^{-4}$	<0·04	Woodford (1951)
Allt Dubhaig	2·3	20–98	$2{\cdot}41 \times 10^{-2}$	$1{\cdot}29 \times 10^{-3}$	<0·04	Ferguson and Ashworth (1991)
Sunwapta River	2·2	0·2–57	$8{\cdot}07 \times 10^{-3}$	$1{\cdot}60 \times 10^{-3}$	<0·04	Dawson (1988)

Table I. Continued

	Length (km)	*D* (mm)	S_0	ε (m^{-1})	*p* (%)	References
White River Sandur	2·1	–	$2{\cdot}4 \times 10^{-1}$	$1{\cdot}5 \times 10^{-3}$	–	Church (1972)
Lewis Sandur	1·0	5·5–19	$4{\cdot}59 \times 10^{-3}$	$2{\cdot}7 \times 10^{-4}$	–	Church (1972)
Peyto Glacier Outwash*	0·94	105–293	$2{\cdot}33 \times 10^{-2}$	$4{\cdot}64 \times 10^{-4}$	<0·04	McDonald and Banerjee (1971)
Bow Glacier Outwash*	0·68	116–224	$2{\cdot}11 \times 10^{-2}$	$3{\cdot}40 \times 10^{-4}$	<0·04	McDonald and Banerjee (1971)
Alluvial fans						
Trail Canyon	16·6	7·4–74	$1{\cdot}33 \times 10^{-1}$	$4{\cdot}76 \times 10^{-5}$	<0·04	Denny (1965)
San Antonio Canyon	16·1	–	$6{\cdot}01 \times 10^{-2}$	$1{\cdot}06 \times 10^{-4}$	<0·04	Krumbein (1937)
Eagle Mountain	15·9	3·4–12	$5{\cdot}51 \times 10^{-2}$	$4{\cdot}35 \times 10^{-5}$	7·8	Denny (1965)
Hanaupa Canyon	14·6	11–45	$9{\cdot}69 \times 10^{-2}$	$2{\cdot}50 \times 10^{-5}$	<0·04	Denny (1965)
Lila C Mine	12·4	2·3–8·0	$8{\cdot}17 \times 10^{-2}$	$1{\cdot}35 \times 10^{-4}$	<0·04	Denny (1965)
Shadow Mountain	10·0	2·3–98	$3{\cdot}05 \times 10^{-1}$	$3{\cdot}13 \times 10^{-4}$	<0·04	Denny (1965)
Alkali Flat	9·2	2·4–52	$2{\cdot}68 \times 10^{-1}$	$2{\cdot}69 \times 10^{-4}$	<0·04	Denny (1965)
Lytle Creek*	8·1	762–2210	$3{\cdot}11 \times 10^{-2}$	$6{\cdot}35 \times 10^{-5}$	8·5	Eckis (1928)
Johnson Canyon	7·6	11–49	$1{\cdot}08 \times 10^{-1}$	$7{\cdot}59 \times 10^{-5}$	<0·04	Denny (1965)
Funeral Peak	7·5	4·0–12·0	$1{\cdot}63 \times 10^{-1}$	$1{\cdot}44 \times 10^{-4}$	<0·04	Denny (1965)
Shadow Mountain	7·3	6·5–28	$1{\cdot}02 \times 10^{-1}$	$2{\cdot}58 \times 10^{-4}$	<0·04	Denny (1965)
Arrow River Canyon*	5·5	434–1133	$3{\cdot}04 \times 10^{-2}$	$3{\cdot}32 \times 10^{-5}$	–	Bluck (1964)
Bat Mountain	5·4	2·6–14	$9{\cdot}10 \times 10^{-2}$	$2{\cdot}14 \times 10^{-4}$	<0·04	Denny (1965)
Deadman Pass	3·9	3·4–15	$2{\cdot}17 \times 10^{-1}$	$5{\cdot}83 \times 10^{-4}$	<0·04	Denny (1965)
Antelope Springs*	3·7	30–884	$1{\cdot}07 \times 10^{-1}$	$4{\cdot}81 \times 10^{-4}$	<0·04	Lustig (1965)
Shadow Mountain	3·4	4·0–30	$1{\cdot}22 \times 10^{-1}$	$4{\cdot}58 \times 10^{-4}$	<0·04	Denny (1965)
Lila C Mine	3·4	2·3–3·4	$2{\cdot}15 \times 10^{-1}$	$4{\cdot}48 \times 10^{-4}$	9·4	Denny (1965)
Copper Canyon	1·9	3·0–9·8	$9{\cdot}34 \times 10^{-2}$	$2{\cdot}61 \times 10^{-4}$	9·4	Denny (1965)
Aubrey Cliffs B*	1·6	111–854	$5{\cdot}25 \times 10^{-2}$	$6{\cdot}08 \times 10^{-4}$	<0·04	Blissenbach (1952)
Aubrey Cliffs A*	0·64	143–900	$1{\cdot}11 \times 10^{-1}$	$2{\cdot}83 \times 10^{-3}$	<0·04	Blissenbach (1952)
Bat Mountain	0·64	4·6–13	$6{\cdot}73 \times 10^{-2}$	$5{\cdot}77 \times 10^{-4}$	6·7	Denny (1965)
Paiute Chute*	0·37	914–2990	$9{\cdot}13 \times 10^{-1}$	$7{\cdot}70 \times 10^{-3}$	<0·04	Lustig (1965)
Paiute Chute*	0·30	884–3170	$6{\cdot}48 \times 10^{-1}$	$6{\cdot}00 \times 10^{-3}$	<0·04	Lustig (1965)
Westgard Pass A*	0·017	91–274	$5{\cdot}59 \times 10^{-1}$	$7{\cdot}35 \times 10^{-2}$	<0·04	Lustig (1965)
Westgard Pass B*	0·014	61–244	$6{\cdot}65 \times 10^{-1}$	$8{\cdot}10 \times 10^{-2}$	<0·04	Lustig (1965)
Tailings deltas						
Denison uranium	3·375	0·050–0·13	$1{\cdot}27 \times 10^{-2}$	$3{\cdot}36 \times 10^{-4}$	<0·04	Conlin (1989)
Pine Point lead-zinc	2·140	sand	$7{\cdot}21 \times 10^{-3}$	$2{\cdot}31 \times 10^{-4}$	<0·04	Conlin (1989)
Uranium 5	0·718	sand	$7{\cdot}11 \times 10^{-2}$	$3{\cdot}31 \times 10^{-3}$	<0·04	Blight *et al.* (1985)
Anamax copper	0·636	sand	$8{\cdot}08 \times 10^{-3}$	$2{\cdot}56 \times 10^{-3}$	<0·04	Smith (1984)
Agnico-Eagle gold	0·630	sand	$1{\cdot}21 \times 10^{-2}$	$2{\cdot}50 \times 10^{-3}$	<0·04	Conlin (1989)
Uranium 4	0·614	sand	$9{\cdot}41 \times 10^{-2}$	$3{\cdot}52 \times 10^{-3}$	<0·04	Blight *et al.* (1985)
Uranium 3	0·606	sand	$9{\cdot}43 \times 10^{-2}$	$3{\cdot}56 \times 10^{-3}$	<0·04	Blight *et al.* (1985)
Uranium 1	0·561	sand	$9{\cdot}01 \times 10^{-2}$	$3{\cdot}51 \times 10^{-3}$	<0·04	Blight *et al.* (1985)
Climax molybdenum 2	0·492	sand	$2{\cdot}98 \times 10^{-2}$	$4{\cdot}52 \times 10^{-3}$	<0·04	Smith (1984)
Oaky Creek coal 2	0·480	0·022–0·087	$1{\cdot}37 \times 10^{-2}$	$3{\cdot}69 \times 10^{-3}$	<0·04	Morris and Williams (1997d)
Climax molybdenum 1	0·344	sand	$3{\cdot}27 \times 10^{-2}$	$1{\cdot}05 \times 10^{-2}$	<0·04	Smith (1984)
Quirke uranium	0·342	sand	$1{\cdot}88 \times 10^{-2}$	$1{\cdot}40 \times 10^{-3}$	<0·04	Conlin (1989)
President Steyn gold	0·315	sand	$1{\cdot}36 \times 10^{-2}$	$1{\cdot}70 \times 10^{-2}$	<0·04	Bentel (1981)
Doyon gold	0·300	sand	$1{\cdot}88 \times 10^{-2}$	$8{\cdot}02 \times 10^{-3}$	<0·04	Conlin (1989)
Climax molybdenum 3	0·295	sand	$1{\cdot}35 \times 10^{-2}$	$3{\cdot}72 \times 10^{-3}$	<0·04	Smith (1984)
Uranium 6	0·293	sand	$8{\cdot}83 \times 10^{-2}$	$2{\cdot}81 \times 10^{-3}$	<0·04	Blight *et al.* (1985)
Meandu coal	0·180	0·018–0·028	$1{\cdot}59 \times 10^{-2}$	$2{\cdot}84 \times 10^{-3}$	<0·04	Morris (1993)

Table I. Continued

	Length (km)	*D* (mm)	S_0	ε (m^{-1})	*p* (%)	References
Platinum A2	0·157	sand	$7{\cdot}80 \times 10^{-2}$	$1{\cdot}76 \times 10^{-2}$	<0·04	Bentel (1981)
Premier diamond	0·150	–	$3{\cdot}44 \times 10^{-2}$	$7{\cdot}87 \times 10^{-3}$	<0·04	Bentel (1981)
Platinum A1	0·115	0·070–0·20	$5{\cdot}49 \times 10^{-2}$	$1{\cdot}72 \times 10^{-2}$	<0·04	Bentel (1981)
Union Carbide uranium	0·082	sand	$4{\cdot}42 \times 10^{-2}$	$5{\cdot}73 \times 10^{-3}$	<0·04	Smith (1984)
Platinum C	0·075	sand	$6{\cdot}44 \times 10^{-2}$	$3{\cdot}54 \times 10^{-2}$	<0·04	Bentel (1981)
Platinum B2	0·074	sand	$1{\cdot}09 \times 10^{-1}$	$2{\cdot}80 \times 10^{-2}$	<0·04	Bentel (1981)
Aberdare coal	0·070	0·066–0·35	$1{\cdot}26 \times 10^{-2}$	$2{\cdot}31 \times 10^{-2}$	<0·04	Morris (1993)
Oaky Creek coal 1	0·066	0·067–0·12	$2{\cdot}12 \times 10^{-2}$	$1{\cdot}67 \times 10^{-2}$	<0·04	Morris and Williams (1997d)
Platinum B1	0·055	sand	$1{\cdot}05 \times 10^{-1}$	$3{\cdot}53 \times 10^{-2}$	<0·04	Bentel (1981)
Hecla silver	0·049	sand	$3{\cdot}36 \times 10^{-2}$	$2{\cdot}41 \times 10^{-2}$	<0·04	Smith (1984)
Platinum D	0·041	sand	$9{\cdot}45 \times 10^{-2}$	$4{\cdot}76 \times 10^{-2}$	<0·04	Bentel (1981)
Pathfinder uranium	0·041	sand	$8{\cdot}20 \times 10^{-2}$	$1{\cdot}81 \times 10^{-2}$	<0·04	Smith (1984)
Alma Mill gold sulphides	0·023	sand	$7{\cdot}71 \times 10^{-2}$	$6{\cdot}31 \times 10^{-2}$	<0·04	Smith (1984)
Coal co-disposal deltas						
Jeebropilly	0·123	3·0–21	$9{\cdot}47 \times 10^{-2}$	$5{\cdot}85 \times 10^{-3}$	<0·04	Morris and Williams (1997c)
Goonyella-Riverside 6	0·030	5·0–20	$2{\cdot}02 \times 10^{-2}$	$8{\cdot}02 \times 10^{-2}$	<0·04	Morris and Williams (1997a)
Goonyella-Riverside 3	0·030	3·5–20	$1{\cdot}80 \times 10^{-1}$	$6{\cdot}57 \times 10^{-2}$	<0·04	Morris and Williams (1997a)
Goonyella-Riverside 4	0·029	5·3–16	$2{\cdot}21 \times 10^{-1}$	$8{\cdot}83 \times 10^{-2}$	<0·04	Morris and Williams (1997a)
Goonyella-Riverside 2	0·024	4·3–13	$2{\cdot}22 \times 10^{-1}$	$1{\cdot}29 \times 10^{-1}$	<0·04	Morris and Williams (1997a)
Warkworth 3	0·023	5·9–15	$2{\cdot}97 \times 10^{-1}$	$9{\cdot}77 \times 10^{-2}$	<0·04	Morris and Williams (1997a)
Goonyella-Riverside 1	0·023	5·0–10	$2{\cdot}06 \times 10^{-1}$	$8{\cdot}37 \times 10^{-2}$	<0·04	Morris and Williams (1997a)
Goonyella-Riverside 7	0·018	3·9–8·4	$2{\cdot}06 \times 10^{-1}$	$1{\cdot}15 \times 10^{-1}$	<0·04	Morris and Williams (1997a)
Laboratory flumes						
Seal 32 hrs	0·035	gravel	$2{\cdot}50 \times 10^{-2}$	$2{\cdot}86 \times 10^{-2}$	<0·04	Seal *et al.* (1997)
Seal 22 hrs*	0·027	21–43	$1{\cdot}78 \times 10^{-2}$	$1{\cdot}05 \times 10^{-2}$	<0·04	Seal *et al.* (1997)
Seal 14 hrs	0·020	gravel	$2{\cdot}04 \times 10^{-2}$	$3{\cdot}46 \times 10^{-2}$	<0·04	Seal *et al.* (1997)
Fan A4	0·00190	sand	$1{\cdot}6 \times 10^{-1}$	1·02	<0·04	Fan (1989)
Fan A2	0·00185	sand	$1{\cdot}36 \times 10^{-1}$	1·05	<0·04	Fan (1989)
Fan A3	0·00172	sand	$1{\cdot}44 \times 10^{-1}$	1·11	<0·04	Fan (1989)
Fan A1	0·00170	sand	$1{\cdot}31 \times 10^{-1}$	1·15	<0·04	Fan (1989)
Fan B3	0·00170	sand	$1{\cdot}73 \times 10^{-1}$	1·17	<0·04	Fan (1989)
Nickel tailings	0·00165	sand	$1{\cdot}22 \times 10^{-2}$	$3{\cdot}23 \times 10^{-1}$	<0·04	
Meandu 1 coal tailings	0·00160	sand	$8{\cdot}18 \times 10^{-3}$	$3{\cdot}94 \times 10^{-1}$	<0·04	Morris and Williams (1996)
Gold tailings	0·00160	sand	$2{\cdot}01 \times 10^{-2}$	$3{\cdot}31 \times 10^{-1}$	<0·04	
Fan B4	0·00160	sand	$1{\cdot}55 \times 10^{-1}$	1·23	<0·04	Fan (1989)
Fan B2	0·00160	sand	$1{\cdot}52 \times 10^{-1}$	1·12	<0·04	Fan (1989)
Aberdare coal tailings	0·00150	sand	$1{\cdot}74 \times 10^{-2}$	1·12	<0·04	Morris and Williams (1996)
Meandu 2 coal tailings	0·00150	sand	$6{\cdot}07 \times 10^{-3}$	$6{\cdot}12 \times 10^{-2}$	<0·04	Morris and Williams (1996)
Oaky Creek 1 coal tailings	0·00150	sand	$5{\cdot}49 \times 10^{-3}$	$4{\cdot}44 \times 10^{-1}$	<0·04	
Oaky Creek 2 coal tailings	0·00150	sand	$4{\cdot}17 \times 10^{-3}$	$4{\cdot}64 \times 10^{-2}$	<0·04	
Gold tailings	0·00140	sand	$6{\cdot}10 \times 10^{-2}$	2·31	<0·04	
Coal tailings	0·00130	sand	$3{\cdot}37 \times 10^{-2}$	$6{\cdot}14 \times 10^{-1}$	<0·04	
Coal tailings	0·00123	sand	$6{\cdot}30 \times 10^{-2}$	2·38	<0·04	
Fan B1	0·00115	sand	$1{\cdot}93 \times 10^{-1}$	1·76	<0·04	Fan (1989)
Gold tailings	0·00110	sand	$6{\cdot}50 \times 10^{-3}$	$1{\cdot}23 \times 10^{-1}$	<0·04	

* Maximum particle size

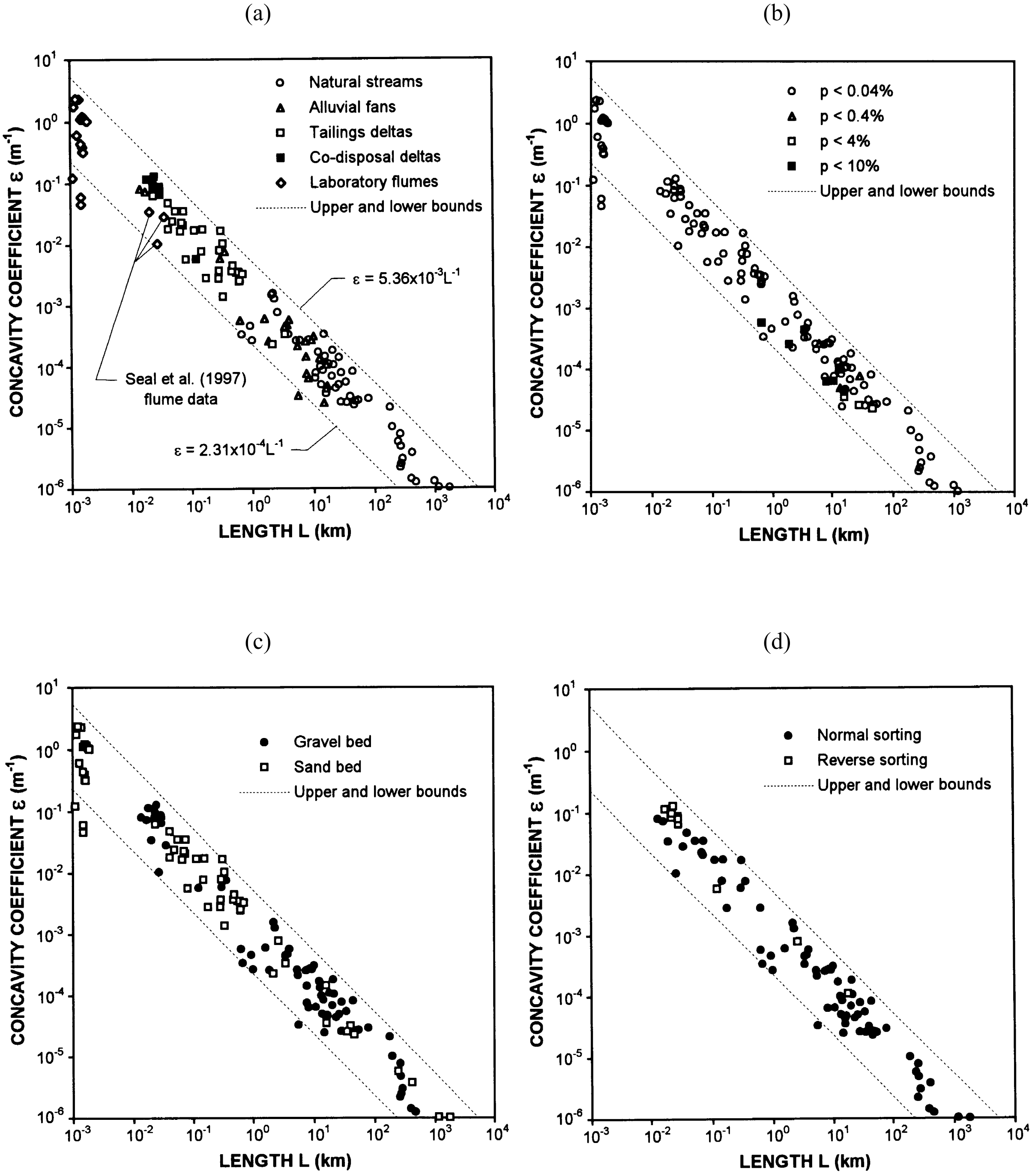

Figure 2. The variation of the profile concavity coefficient ε with the stream segment length L showing (a) worldwide subaerial aqueous flows, and the distributions of (b) the level of significance p for each data point, (c) sand and gravel bed streams, and (d) normal and reverse sorting

the median, D_{50}, and maximum, D_{max}, particle sizes, respectively. The division into normal and reverse sorting is mostly based on the exponential sorting data presented in Table I of Morris and Williams (1999).

In Figure 2a to c, there are no obvious trends in the data relative to the upper and lower bounds. Similar results were obtained for the distributions of the average and maximum bed slopes, which ranged in magnitude from $5{\cdot}6 \times 10^{-5}$ to $4{\cdot}6 \times 10^{-1}$ and $1{\cdot}1 \times 10^{-4}$ to $9{\cdot}1 \times 10^{-1}$, respectively. Hence the data set appears to be homogeneous with respect to all of these parameters. The reverse sorting data points are too few (Figure 2d) to enable a conclusion to be reached regarding their homogeneity.

For the tailings deltas, co-disposal deltas and laboratory flume tests included in the L,ε data set, limited data on the input C, discharges Q, and particle size distributions were available (48, 33 and 34 data points, respectively, including the three outliers). The C and Q values and the ratios D_{75}/D_{25} of the upper and lower quartiles of the particle size distributions ranged over 0·0018–0·56, 0·05–0·14 m^3s^{-1} and 1·35–70, respectively.

The C, Q and D_{75}/D_{25} data points cover the whole range between the upper and lower bounds and, although they are limited to L less than about 2·2 km, show no apparent trends with magnitude relative to the bounds. Hence, on the basis of the limited available data, the L,ε data set appears to be homogeneous with respect to these parameters also.

While flow in natural streams, on coal co-disposal deltas, and in gravel bed flumes and irrigation canals is almost always turbulent, flows on tailings deltas are not infrequently arranged to be laminar. There are no biases evident in the turbulent flow data, and the laminar flow data points are too few for any associated biases to be detected.

Overall, the L,ε data appear to be homogeneous with respect to many of the parameters and phenomena relevant to profile concavity, with the possible exception of reverse sorting. Hence, with the exception of the four outliers, they may reasonably be treated as a single data set.

Worldwide correlation of ε with α

Figure 3a, a plot of ε versus α for the worldwide data from a wide range of subaerial aqueous flows, shows that they conform to:

$$\varepsilon = E\alpha \tag{5}$$

where E is a constant. For these data, L ranges from 14 m for an alluvial micro-fan to 1770 km for the Mississippi River. The associated R and p values are −0·975 and less than 0·04 percent, respectively, again indicating a very strong correlation. This is unsurprising, given that both α (Morris and Williams, 1999) and ε are strongly correlated with L, and the α,L and ε,L data sets on which these correlations are based have high commonality (about 50 percent).

The width of the data band defined by the upper and lower bounds shown in Figure 3 is 1·14 orders of magnitude, and the geometric mid-range value of E is 1·36. The lower bound, which is well defined over the whole range of α, corresponds to the lower limit of the data points with fully quantified p values (Figure 3a and b). The two outlying data points correspond to the Lewis Sandur and the Arrow River Canyon alluvial fan (Table I). The R and p values for the data set excluding the two outliers are −0·980 and less than 0·04 percent, respectively.

Homogeneity of the α, ε data set

The distributions of the p for the individual α,ε data points, of sand and gravel bed streams, and of normal and reverse sorting are shown in Figure 3b,c and d, respectively. The range of D is identical with that for the L,ε data.

There are no evident trends in the data relative to the upper and lower bounds either in Figure 3a to d or for the equivalent scatter diagrams for D_{50} and D_{max}, and the average and maximum S. Hence the data set appears to be homogeneous with respect to all of these parameters.

Very limited input C, Q and D_{75}/D_{25} data (13, nine and five data points, respectively) for the tailings deltas, co-disposal deltas and laboratory flume tests are also available. These data points, whose α values are all

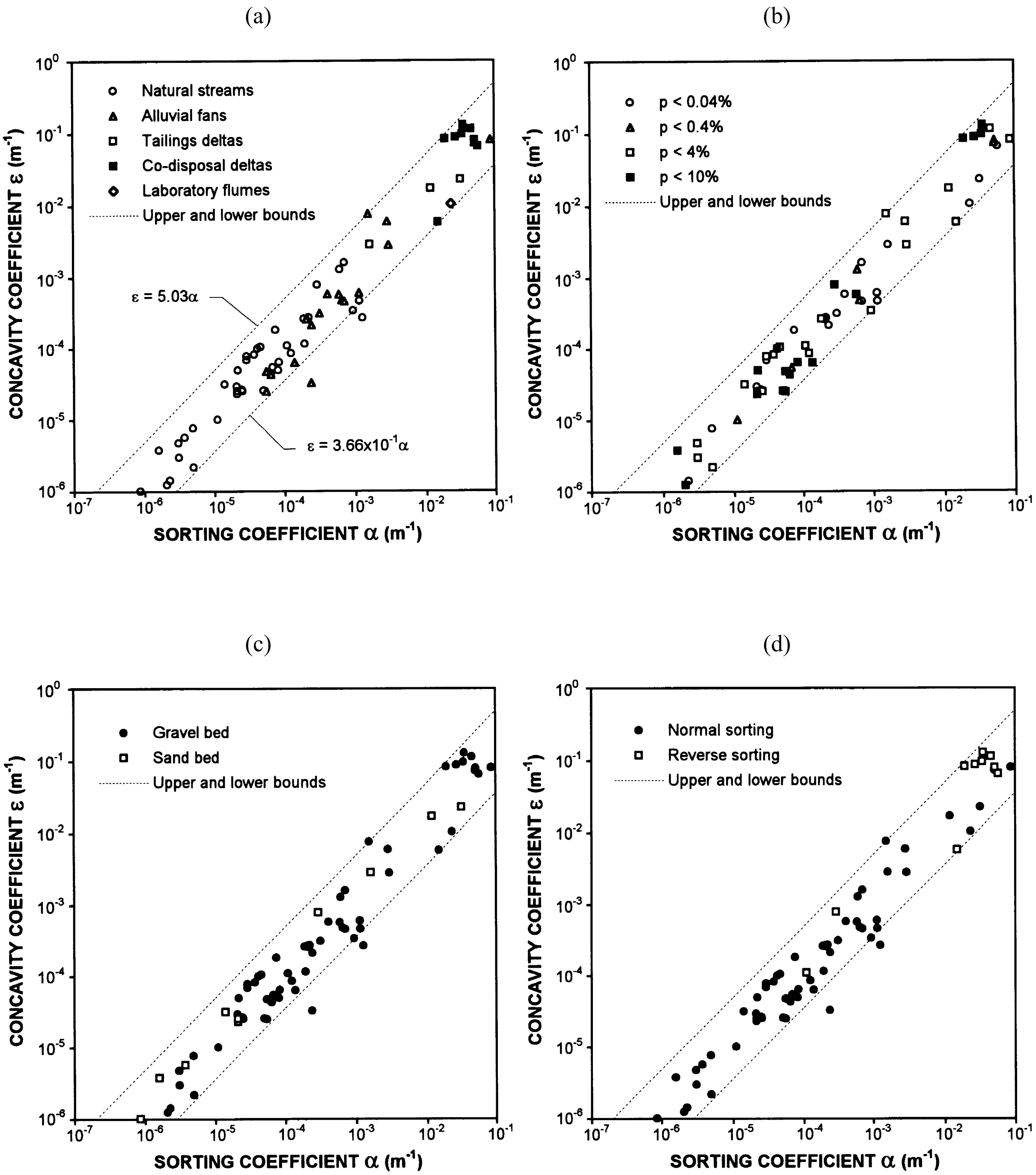

Figure 3. The variation of the profile concavity coefficient ε with the particle size diminution coefficient α showing (a) worldwide subaerial aqueous flows, and the distributions of (b) the level of significance p for each data point, (c) sand and gravel bed streams, and (d) normal and reverse sorting

Table II. Values for canals and natural streams of the median bed sediment particle size D_{50} and the coefficient E in Equation 5

Reference	D_{50} (mm)	Mean E	Range of E
Field data			
Schoklitsch in Shulits (1936)	–	1·00	–
Henderson (1963)	>6	1·14	–
Hey and Thorne (1986)	14–176	0·71	–
Yalin (1992)	6–176	0·99	0·59–1·29
Theoretical			
Sternberg (1875) in Shulits (1936)	–	0·50	–
Lacey (1946)	0·01–110	–	0·40–1·71
Lane (1955)	0·01–110	–	0·37–1·51

greater than about $1{\cdot}6 \times 10^{-3}$, are well distributed between the upper and lower bounds, and show no apparent trends with magnitude. Hence, on the basis of the limited available data, the α,ε data set appears to be homogeneous with respect to these parameters also. No conclusions are possible in respect of the minimal laminar flow data.

Overall, the α,ε data appear to be homogeneous with respect to many of the related mobile bed flow parameters and phenomena. Hence they too may reasonably be treated as a single set.

Values of E from other sources

Values of E (Equation 5) from various sources are listed in Table II. All are based on Sternberg's (1875) exponential abrasion law. Schoklitsch's (Shulits, 1936) mean E value estimate of unity was based on limited field data that showed that S is proportional to the bed D in streams with exponential profiles. The other field-data based E values presented in Table II were derived from empirical power law slope-prediction equations in which D appears. Henderson's (1963) and Hey and Thorne's (1986) slope-prediction equations were respectively based on data from 33 straight and meandering rivers in the United States and India, and from 52 stable, mobile bed rivers in the United Kingdom. Yalin (1992) compiled 10 slope-prediction equations derived by other researchers for numerous regime canals, rivers and small streams in alluvium.

Sternberg's (1875) theoretical E value is based on the assumptions that flow is two-dimensional and that the stream width is proportional to the discharge. The former assumption is reasonable (Morris and Williams, 1997b), but the latter is arbitrary.

Lacey (1946) and Lane (1955) present equations describing flow in stable regime (mobile bed) channels and immobile bed channels, respectively, from which power law slope-prediction equations can be derived. To obtain these equations, it is necessary to eliminate the hydraulic radius, H, from the flow equations. For many subaerial aqueous flows a simple power law relationship exists between Shields' entrainment function and the dimensionless particle size (Novak and Nalluri, 1974; Carling, 1983; Morris and Williams, 1997b). On the basis of the reasonable assumption that, for any given stream segment, that part of H associated with form drag due to ripples or dunes on the bed is a constant fraction of the total H (Morris and Williams, 1997b), it is readily shown that:

$$H = C_1 D^{1+b} S^{-1} \tag{6}$$

where C_1 is a constant over limited ranges of D, and b is a dimensionless constant that ranges in value from $-0{\cdot}64$ to $0{\cdot}29$ for bed particles ranging in size from silt to boulders (van Rijn, 1984; Wang and Shen, 1985). Since this range of b values comprises a large subset of those for all mobile bed subaerial aqueous flows with low relative hydraulic roughness (van Rijn, 1984; Yalin, 1992; Morris and Williams, 1997b), the ranges of E presented in Table II that are based on them and Equation 6 are moderately conservative for such flows.

Discussion of E values and data band widths

The range of E and the geometric mid-range value for the present α,ε data set are 0·366–5·03 and 1·36, respectively, and the width of the data band is 1·14 orders of magnitude (Figure 3a). Except for Sternberg's mean E value of 0·5, with its arbitrary basis, the mean E values listed in Table II are reasonably consistent with each other. However, they are significantly lower than the mid-range E value for the present data set. Also, the field data and theoretically based ranges of E listed in Table II correspond to data band widths of 0·34 and 0·66 orders of magnitude, respectively, significantly smaller than that for the present data set.

A wide range of mobile bed flow types is represented in the present α,ε data set (Table I), whereas the E values presented in Table II apply to a more restricted range of perennial natural streams and large irrigation canals with either zero or bank-full discharges. Both sand and gravel bed streams are represented in Table II, but the relative hydraulic roughness is generally small. For the comparable natural streams included in the present data set, the range and the geometric mid-range value of E set are 0·441–2·65 and 1·08, respectively, and the data band width is 0·78 orders of magnitude. Except for the upper bound E value, these and the corresponding Table II values are reasonably comparable. Moreover, the discrepancy between the upper bound E values is considerably reduced. Hence it is probable that the discrepancies between the E values and data band widths for the present ε,α data set and the Table II data are primarily attributable to differences in the range of hydraulic conditions and, possibly, sediment properties represented.

The application of first-order error propagation theory (Gardner and Ely, 1967) to the widths of the L,α (Morris and Williams, 1999) and L,ε data bands gives an α,ε band width of about 1·70 orders of magnitude. This is significantly greater than the band width of 1·14 obtained for the α,ε data (Figure 3). Given the high commonality of the α,L and ε,L data sets, this implies that much of the scatter shown by these two data sets may be attributable to the same causes.

The width of the data band for the L,ε correlation (1·37 orders of magnitude) is significantly greater than that for the L,α correlation (1·04 orders of magnitude). This is consistent with the dependence of S on D, rather than the reverse, in mature streams flowing through their own sediments (White, 1946).

Since each individual L,ε data point represents a strong correlation, the effect of measurement errors on the scatter is likely to be small. The laboratory flume data of Seal *et al.* (1997) are an exception (Figure 2a). These data, which represent successive stages of the development of the bed profile during a single test with constant inputs, are scattered over about 0·52 orders of magnitude. This occurred at least partly because the maximum input D (64 mm) was approximately equal to the maximum concavity, so that individual sediment particles significantly affected the measured profile.

In most cases, however, the scatter of the L,ε data, like that of the L,α data (Morris and Williams, 1999), is mostly attributable to differences in hydraulic conditions and sediment properties. The homogeneity of the L,ε and α,ε data sets implies that the parameters investigated contribute little to the scatter. However, the lithology and particle size distribution of the bed sediments (Morris and Williams, 1999), the discharge magnitude, and the relative hydraulic roughness may all contribute significantly (White, 1946; Lacey, 1946; Yalin, 1992).

For the L,α data, a band width of about 0·53 orders of magnitude is accounted for by the lithology of the sediments and lateral sediment inflows (Morris and Williams, 1999). The data band width of 0·66 orders of magnitude obtained for the Table II α,ε data is based on data from laboratory tests in which neither lithology nor lateral sediment inflows was relevant. Hence these bands represent independent effects and may be combined, using first-order propagation of errors theory, to give an α,ε band width of 0·85 orders of magnitude. That is, they may together account for up to about 75 per cent of the α,ε data band width of 1·14 orders of magnitude. However, the identification and quantification of the parameters and phenomena contributing to the scatter of the data remain very incomplete.

CONCLUSIONS

There is a very strong worldwide correlation between the exponential bed concavity coefficient, ε, for a wide range of subaerial mobile bed aqueous flows and the stream length, L (Figure 2a), that is complementary to the corresponding correlation for the exponential particle size diminution coefficient, α (Morris and

Williams, 1999). A similar, very strong correlation between ε and α has also been shown to exist. All three correlations extend over virtually the whole range of L, stream bed sediment particle sizes, and some other hydraulic parameters and conditions found on Earth. The two new correlations tend to confirm the earlier suggestion (Morris and Williams, 1999) that there are underlying mechanisms (Pizzuto, 1992; Sinha and Parker, 1996; Morris and Williams, 1997b) common to all kinds of mobile bed subaerial aqueous flows with exponential profiles and particle size diminution.

The α,ε data and correlation are consistent with early theoretical correlations (Sternberg, 1875; Schoklitsch cited in Shulits, 1936) and later data from regime flows. However, the scatter of the data, though relatively small for all three correlations between α, ε and L, is very significant. It is mostly attributable to variations in hydraulic conditions and sediment properties rather than to errors of measurement. The magnitudes of the L,ε and L,α data band widths are consistent with White's (1946) view that the slope of a mature stream is a function of the bed particle size. Lateral sediment inflows and the lithology and particle size distributions of the bed sediments contribute significantly to the scatter of the L,α data (Morris and Williams, 1999), the α,ε data, and probably the L,ε data. Other significant parameters and phenomena remain to be identified, but it has been shown that some parameters are unlikely to contribute significantly.

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